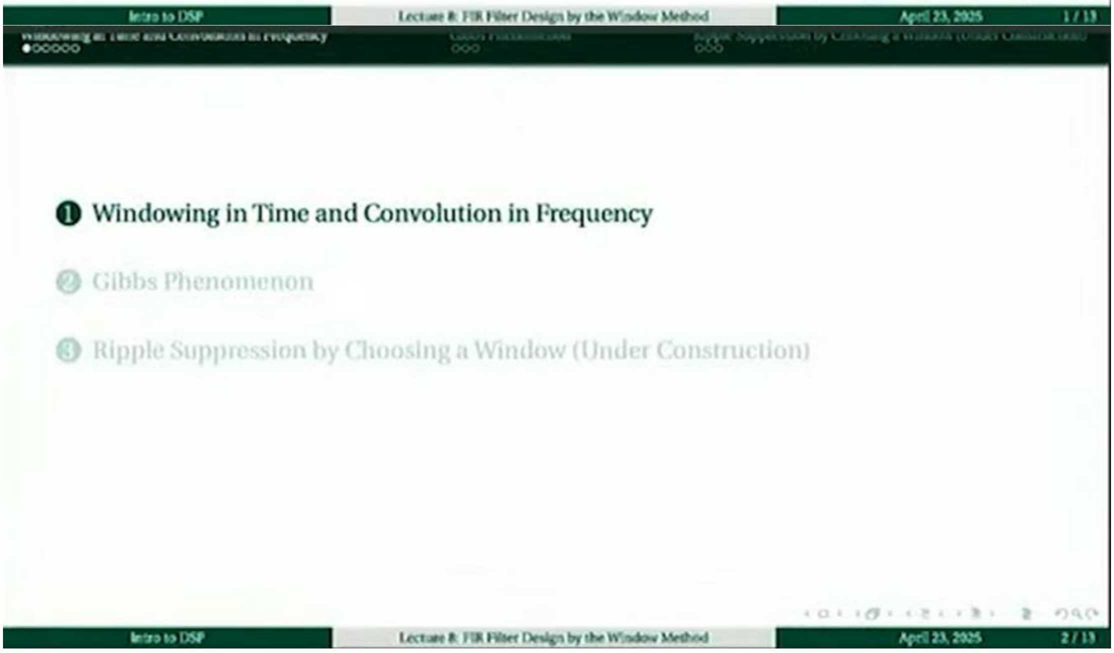
Lecture8 FIR Filter Design by the Window Method



FIR overview: the first step

The idea is to approximate the ideal filter, but make its length finite. In Homework 2 Problem 1, you have seen that

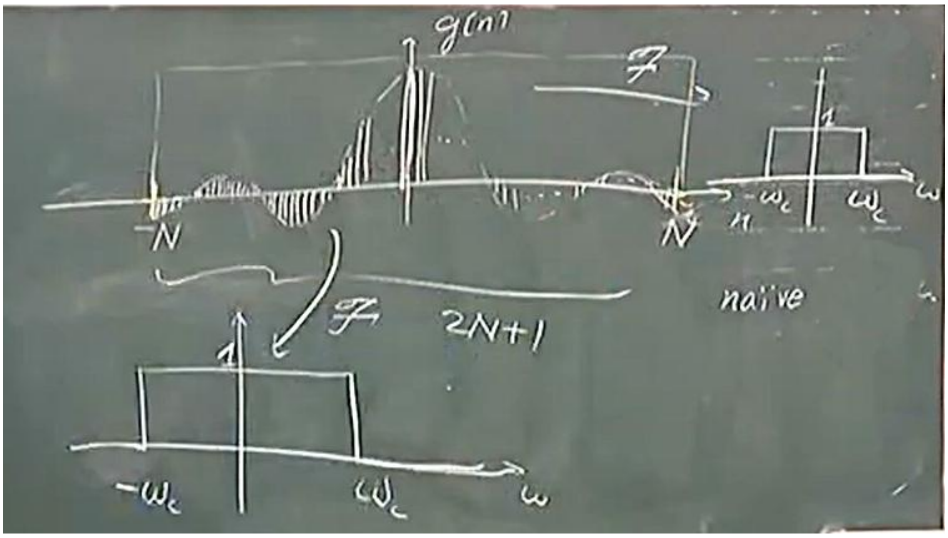
$$g[n] = \begin{cases} \frac{\sin \omega_c n}{\pi n}, & n \neq 0 \\ \frac{\omega_c}{\pi}, & n = 0 \end{cases}$$

is the impulse response for an ideal low-pass filter with a cutoff frequency at ω_c.

Then, truncate $g[n]$ at $-N \le n \le N$; that is, let's define

$$h_N[n] = \begin{cases} g[n], & -N \le n \le N \\ 0, & \text{elsewhere.} \end{cases}$$

This can already be called a "design"! Let's examine its performance in the frequency domain.



Multiplication in time, convolution in frequency

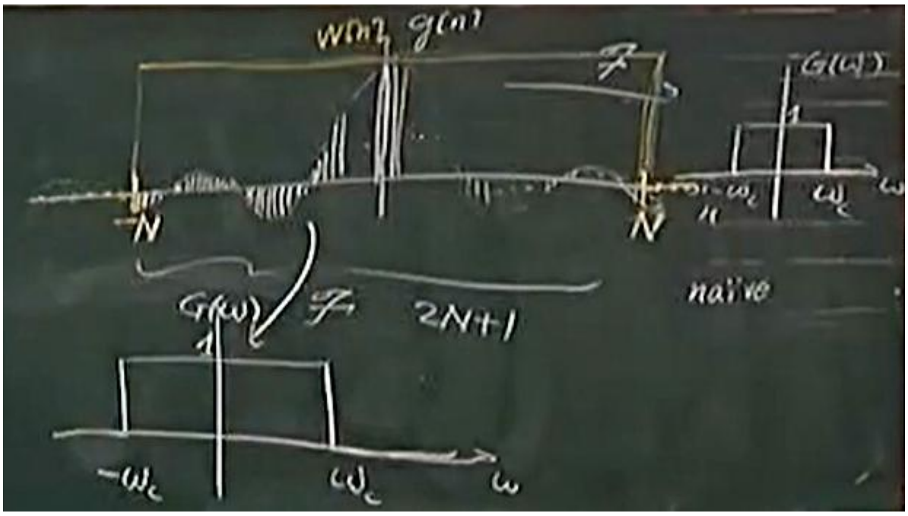
In the previous page, our design can be expressed as $h_N[n] = g[n]w[n]$, where $w[n] = 1, -N \le n \le N$ is called a *rectangular window* of length $2N+1$ (with $w[n] = 0$ elsewhere).

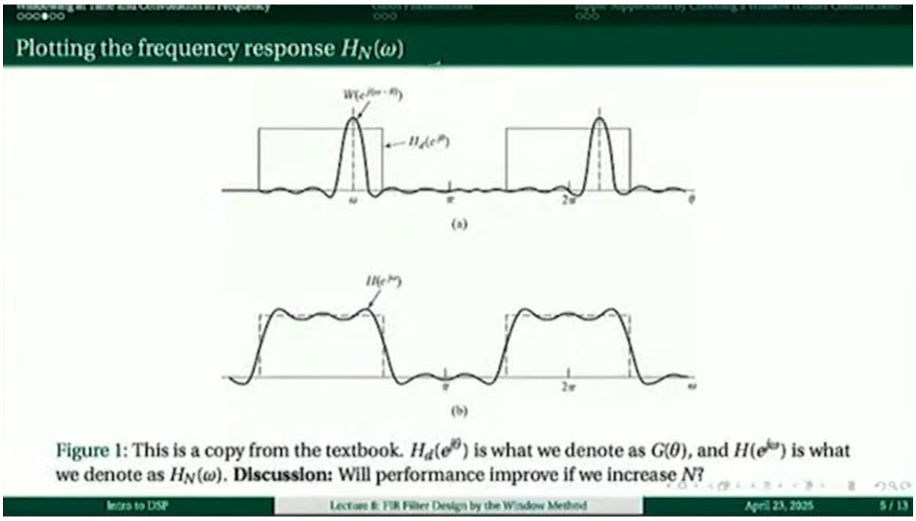
Therefore, the Fourier transform has the following expression

$$H_N(\omega) = G(\omega) \circledast W(\omega) = \frac{1}{2\pi}\int_{-\pi}^{\pi} G(\theta)W(\omega - \theta)\,du$$

Remarks:

1. For simplicity, we write ω here instead of $e^{j\omega}$.
2. Note that both $G(\omega)$ and $W(\omega)$ are real-valued, making it easier to plot the frequency response next.


w[n] g[n]
G(ω)
-N
N
-ωc
ωc
ω
naive
G(ω)
2N+1
-ωc
ωc
ω


Plotting the frequency response $H_N(\omega)$
(a)
(b)
Figure 1: This is a copy from the textbook. $H_d(e^{j\theta})$ is what we denote as $G(\theta)$, and $H(e^{j\omega})$ is what we denote as $H_N(\omega)$. **Discussion:** Will performance improve if we increase N?
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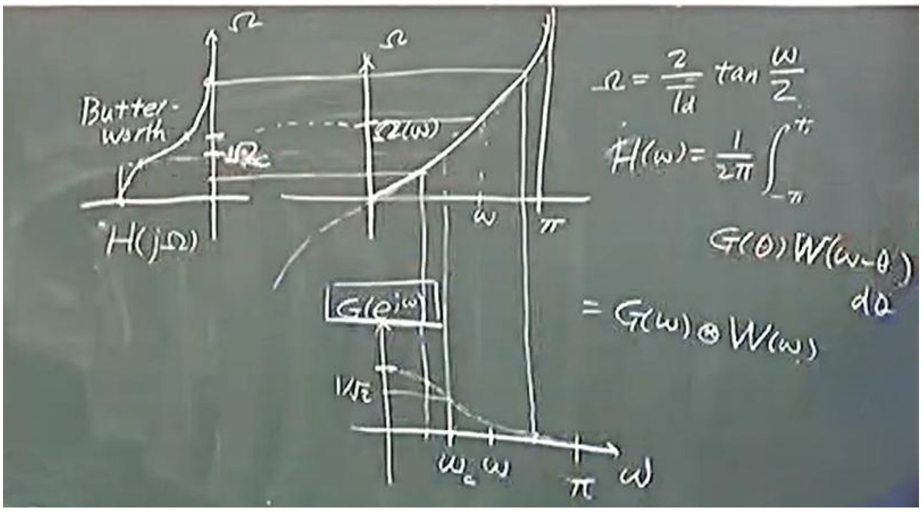
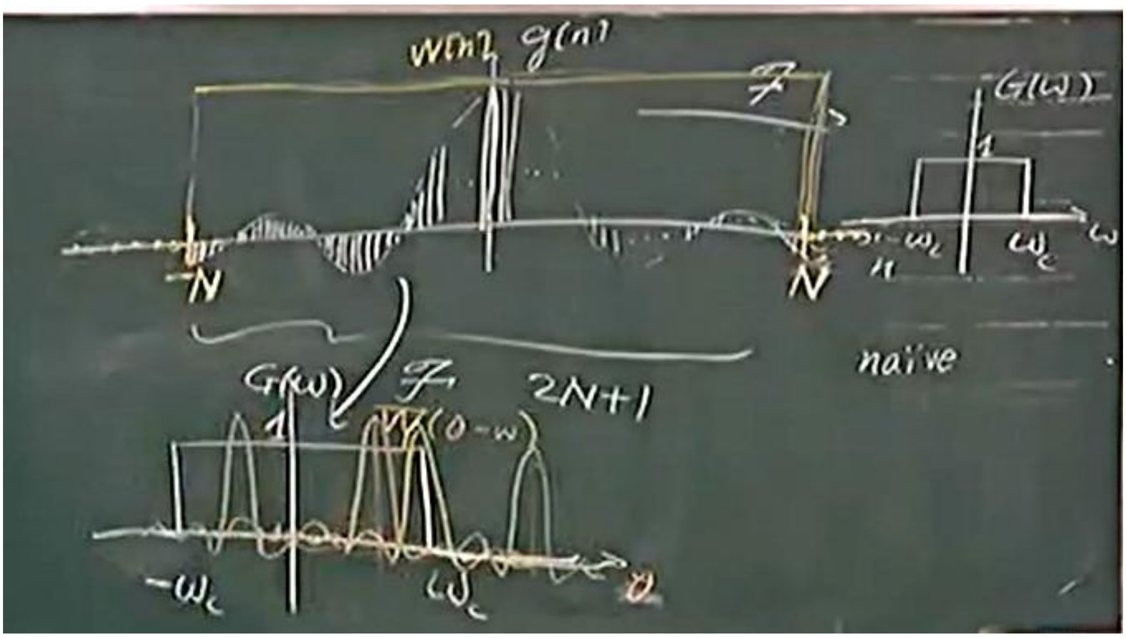

Butterworth
H(jΩ)
Ω(ω)
ω
π
$\Omega = \frac{2}{T_d} \tan \frac{\omega}{2}$
$H(\omega) = \frac{1}{2\pi} \int_{-\pi}^{\pi} G(\theta) W(\omega - \theta) \, d\theta$
$= G(\omega) \circledast W(\omega)$
G(e^{jω})
1/√ε
ωc
ω
π
ω


w(n) g(n)
G(ω)
1
-N
N
n
-ω_c
ω_c
ω
naive
G(ω)
2N+1
W(θ-ω)
-ω_c
ω_c
θ