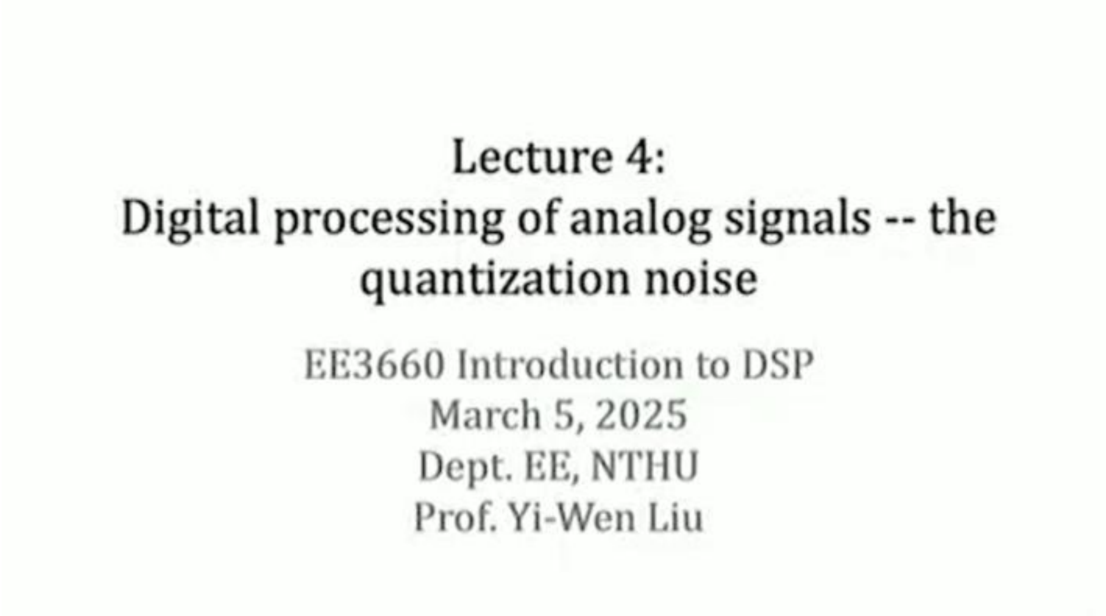
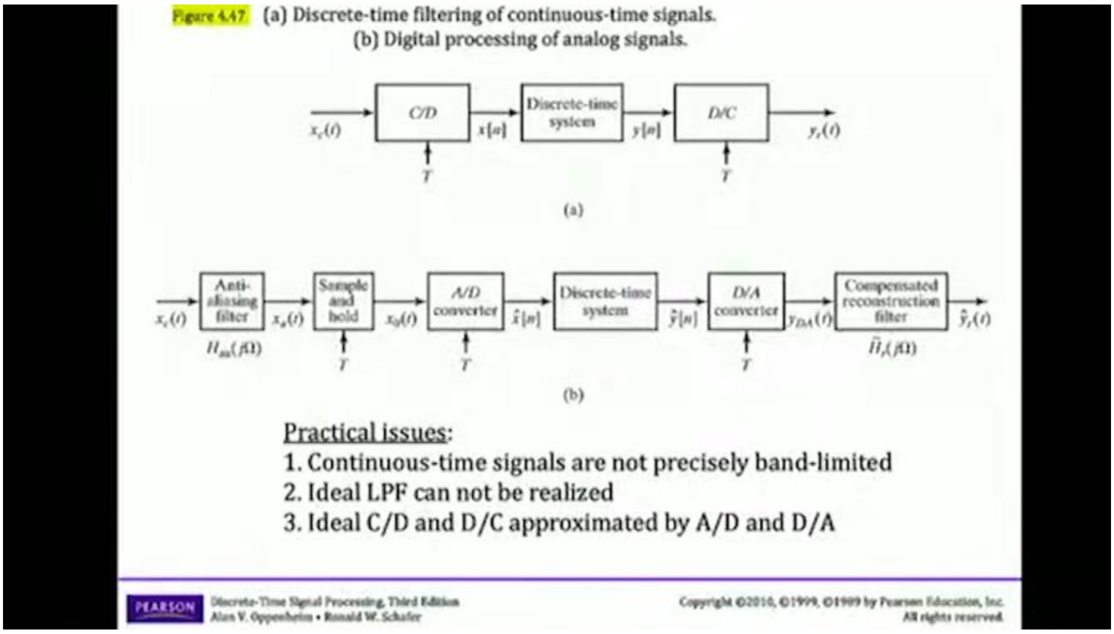
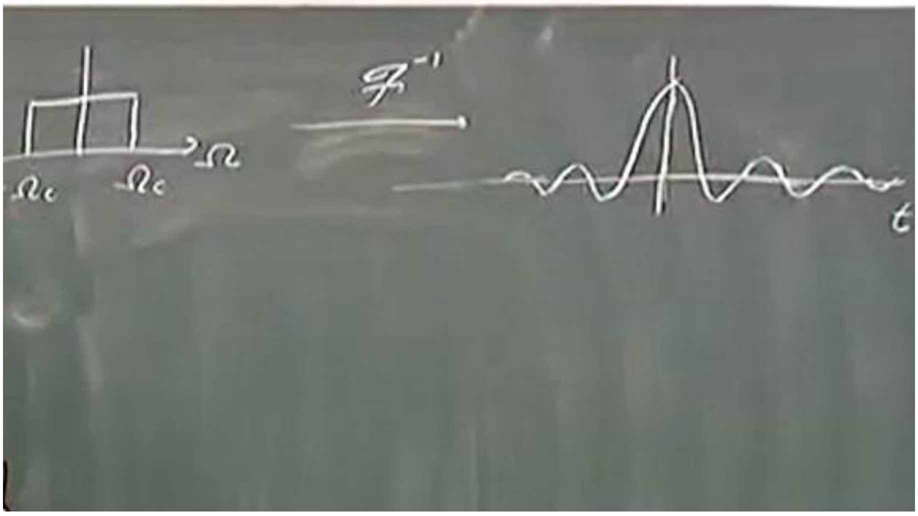
Lecture4 Digital processing of analog signals -- the quantization noise

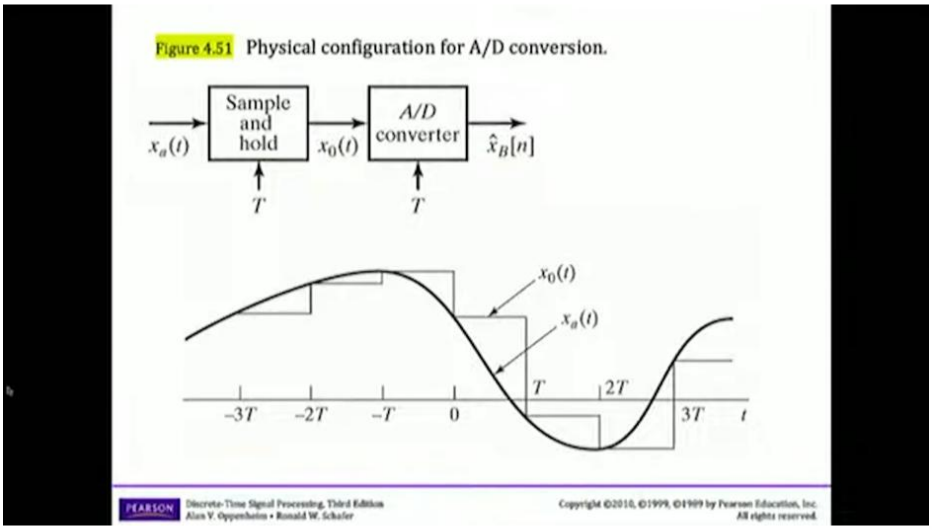


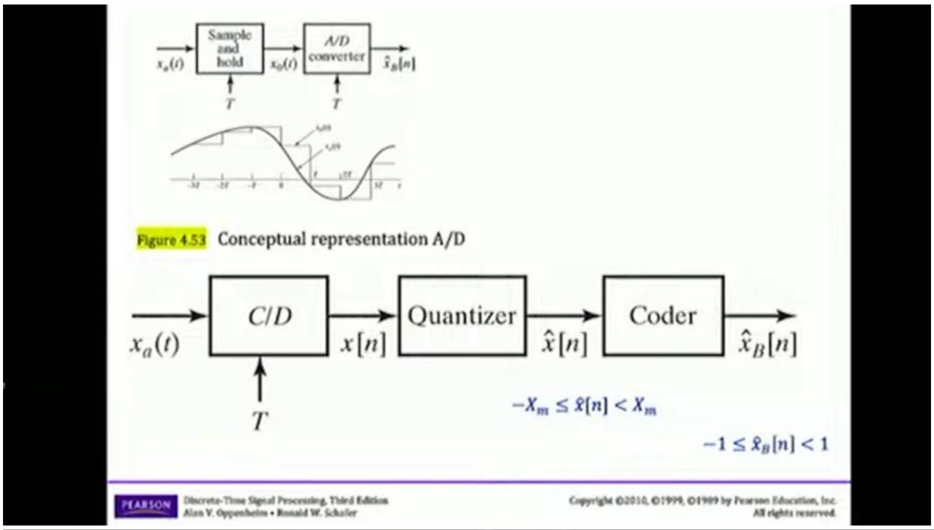




Outlines

- Sample-and-hold
- Quantization and noise
  - Binary representation
  - Noise distribution
  - Noise spectrum




Sample and hold
A/D converter
$x_a(t)$
$x_0(t)$
$\hat{x}_B[n]$
T
Figure 4.53 Conceptual representation A/D
C/D
Quantizer
Coder
$x_a(t)$
$x[n]$
$\hat{x}[n]$
$\hat{x}_B[n]$
T
$-X_m \le \hat{x}[n] < X_m$
$-1 \le \hat{x}_B[n] < 1$
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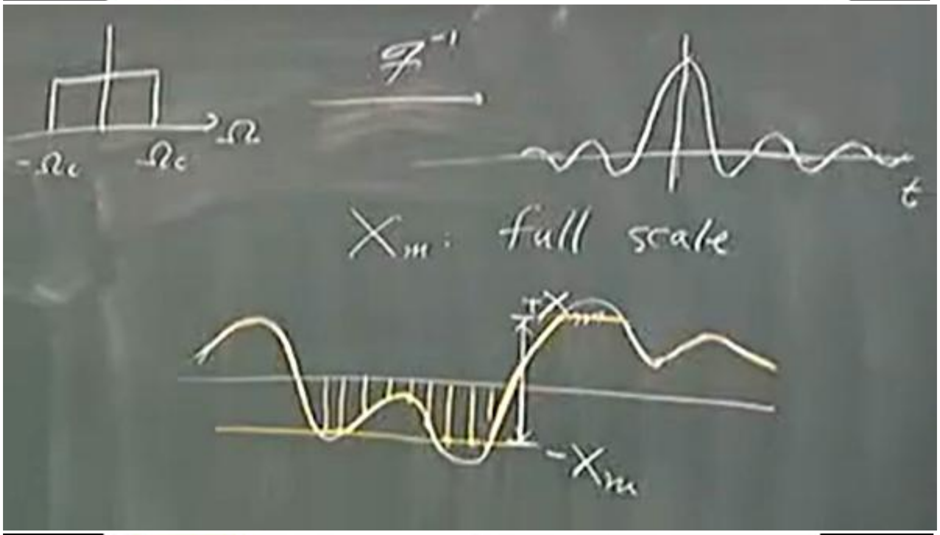
$-\Omega_c$
Ω_c
Ω
$\mathcal{F}^{-1}$
t
X_m: full scale
$-X_m$

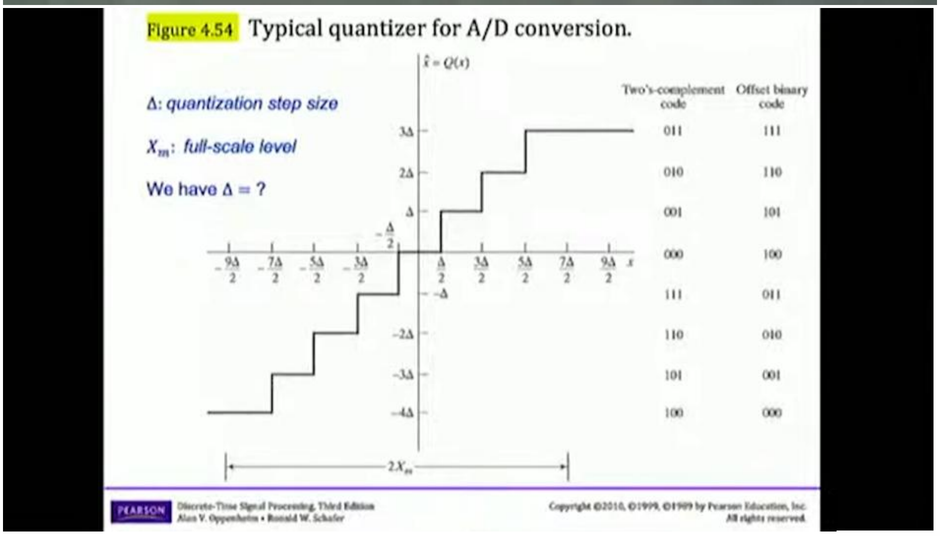
Figure 4.54 Typical quantizer for A/D conversion.
$\hat{x} = Q(x)$
Δ: quantization step size
X_m: full-scale level
We have Δ = ?
Two's-complement code
Offset binary code
011 111
010 110
001 101
000 100
111 011
110 010
101 001
100 000
$2X_m$
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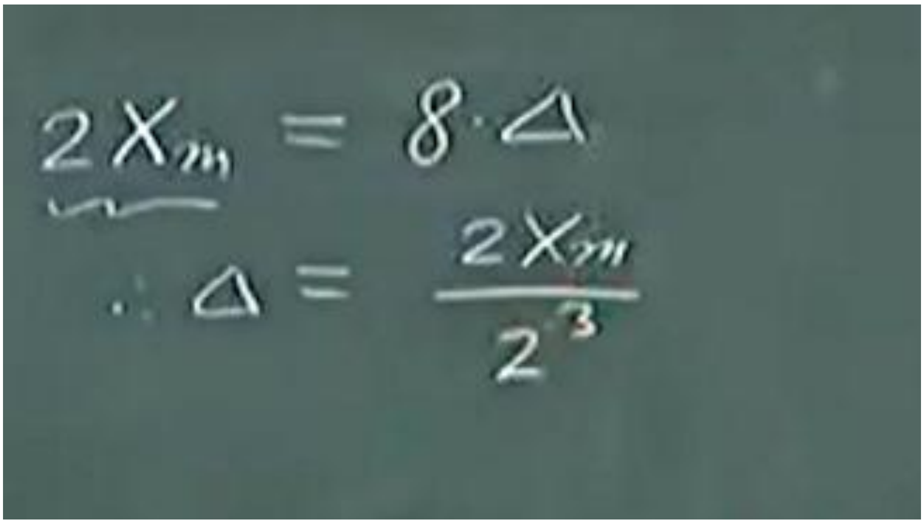
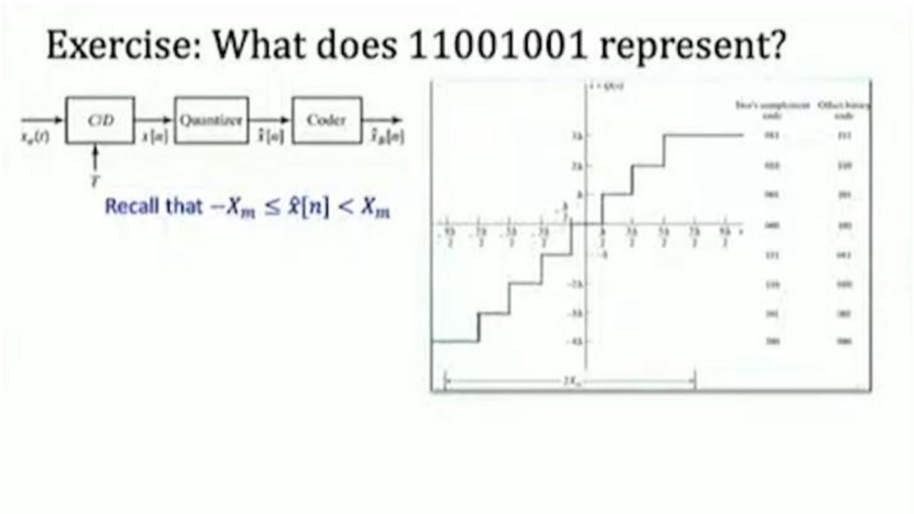
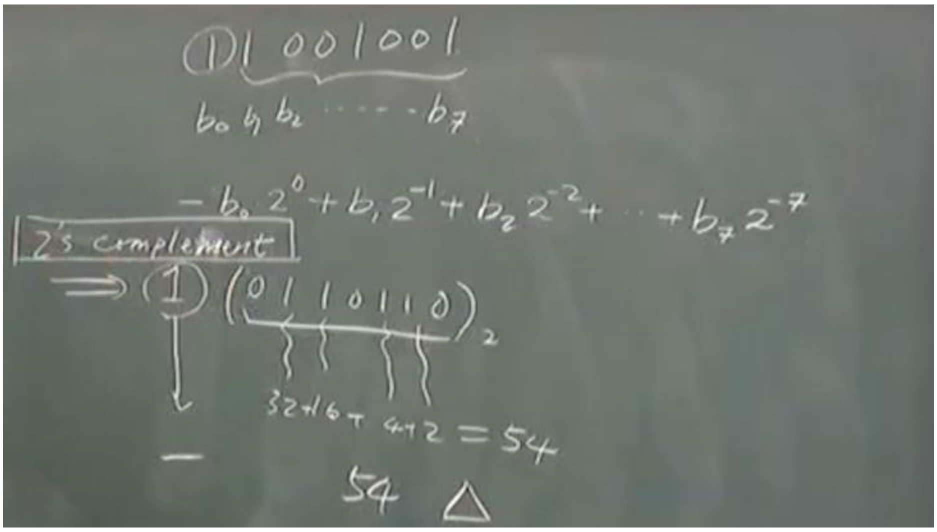

2Xm = 8·Δ
∴ Δ = 2Xm / 2^3


Exercise: What does 11001001 represent?
C/D
Quantizer
Coder
Recall that −Xm ≤ x̂[n] < Xm


①1001001
b0 b1 b2 ····· b7
−b0 2^0 + b1 2^-1 + b2 2^-2 + ··· + b7 2^-7
2's complement
⟹ ① (0110110)2
32+16+4+2 = 54
−
54 Δ

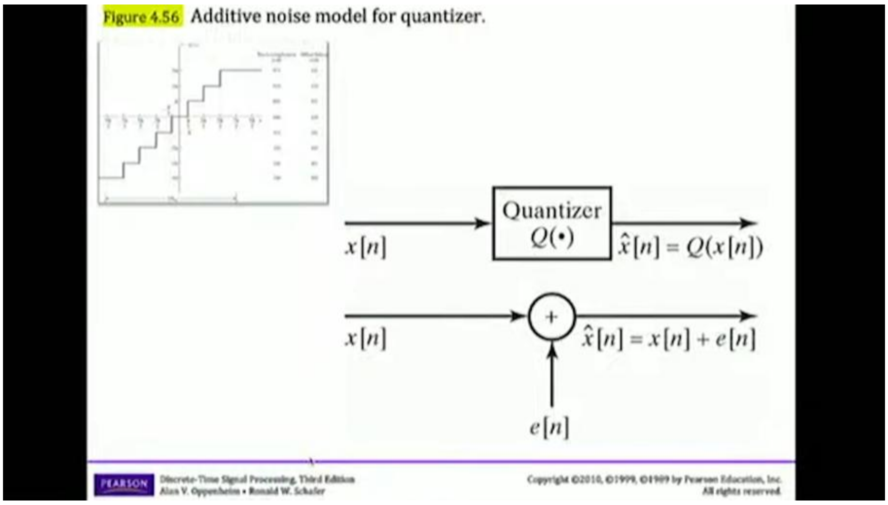


Figure 4.56 Additive noise model for quantizer.
Quantizer
Q(•)
x[n]
x̂[n] = Q(x[n])
x[n]
+
x̂[n] = x[n] + e[n]
e[n]
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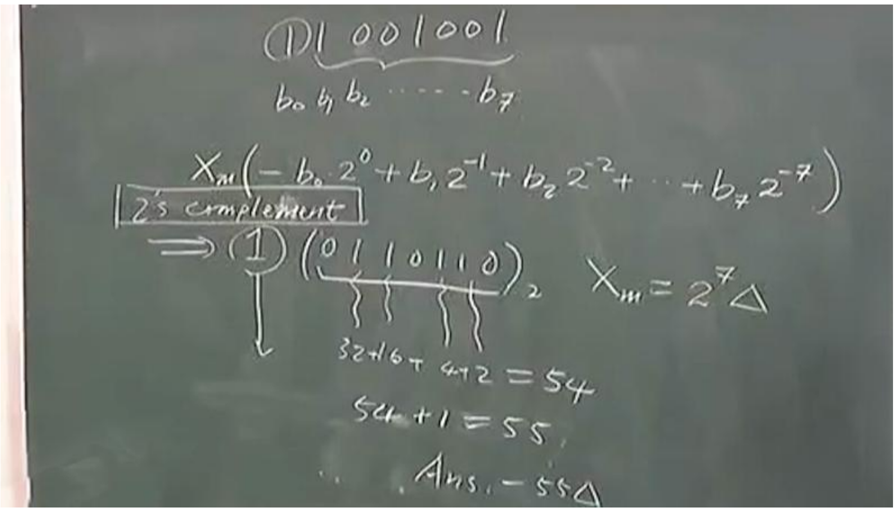
(1)1001001
b0 b1 b2 ····· b7
Xm(− b0·2^0 + b1 2^-1 + b2 2^-2 + ··· + b7 2^-7)
2's complement
⟹ (1) (0110110)2   Xm = 2^7 Δ
32+16+4+2 = 54
54+1 = 55
Ans. −55Δ

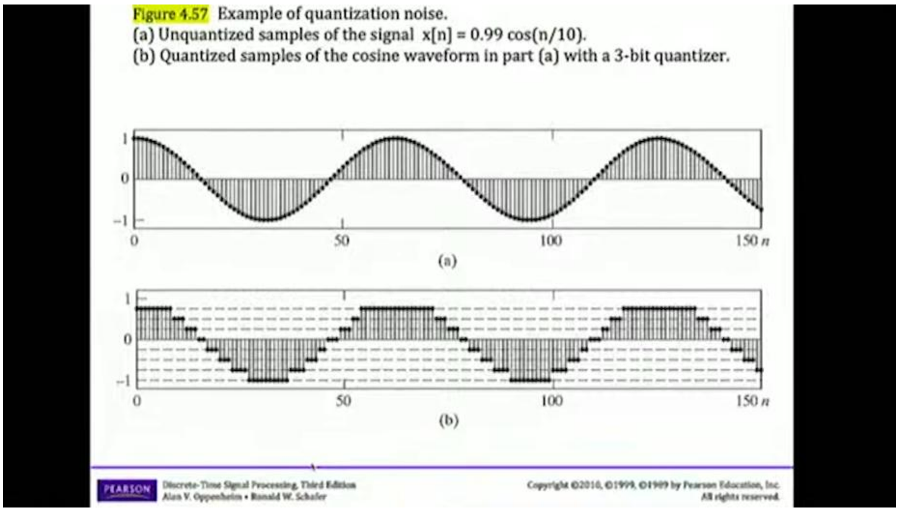
Figure 4.57 Example of quantization noise.
(a) Unquantized samples of the signal x[n] = 0.99 cos(n/10).
(b) Quantized samples of the cosine waveform in part (a) with a 3-bit quantizer.
1
0
−1
0
50
100
150 n
(a)
1
0
−1
0
50
100
150 n
(b)
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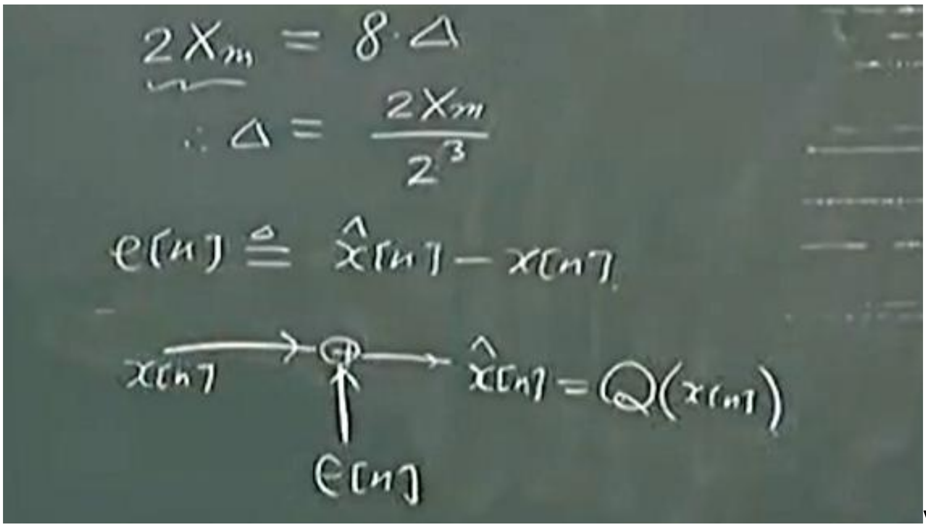
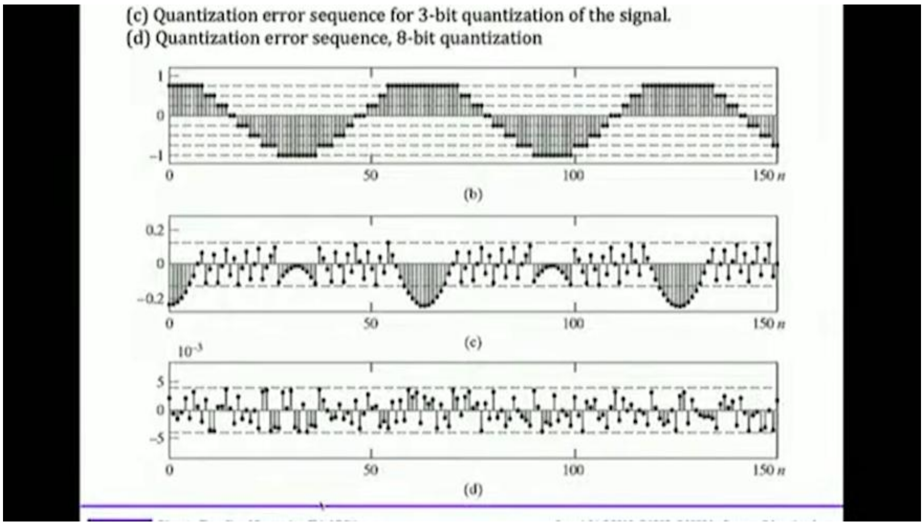

$2X_m = 8\cdot\Delta$
$\therefore \Delta = \frac{2X_m}{2^3}$
$e[n] \triangleq \hat{x}[n] - x[n]$
$x[n]$
$\hat{x}[n] = Q(x[n])$
$e[n]$

v


(c) Quantization error sequence for 3-bit quantization of the signal.
(d) Quantization error sequence, 8-bit quantization
(b)
(c)
(d)
10^{-3}
n

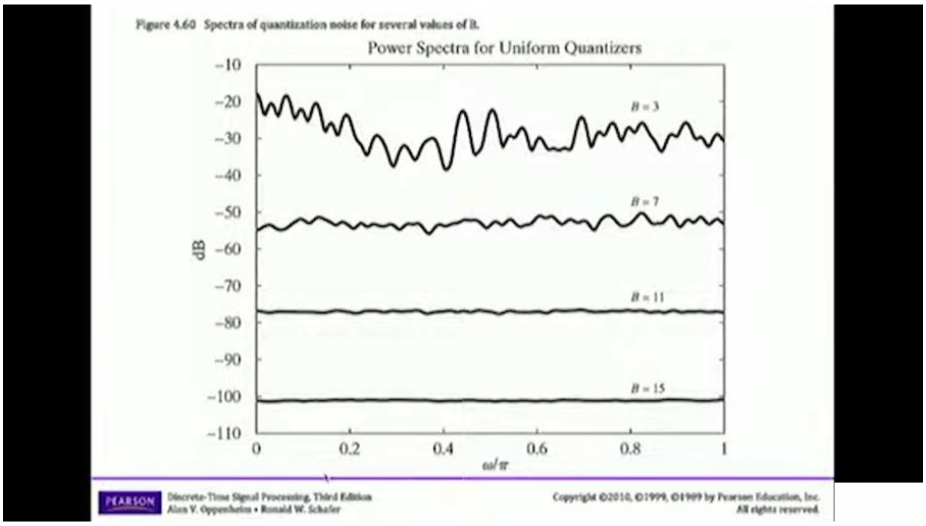


Figure 4.60 Spectra of quantization noise for several values of B.
Power Spectra for Uniform Quantizers
dB
ω/π
B = 3
B = 7
B = 11
B = 15
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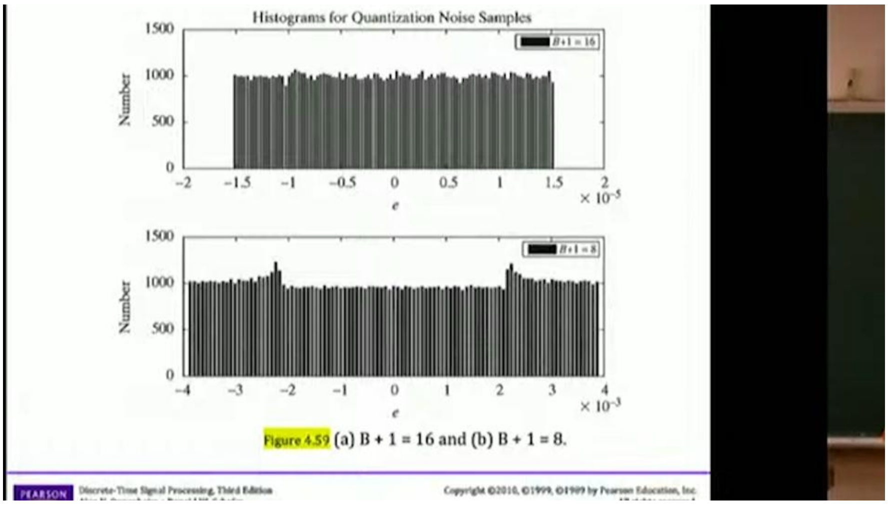
Histograms for Quantization Noise Samples
Number
e
B+1 = 16
B+1 = 8
Figure 4.59 (a) B + 1 = 16 and (b) B + 1 = 8.
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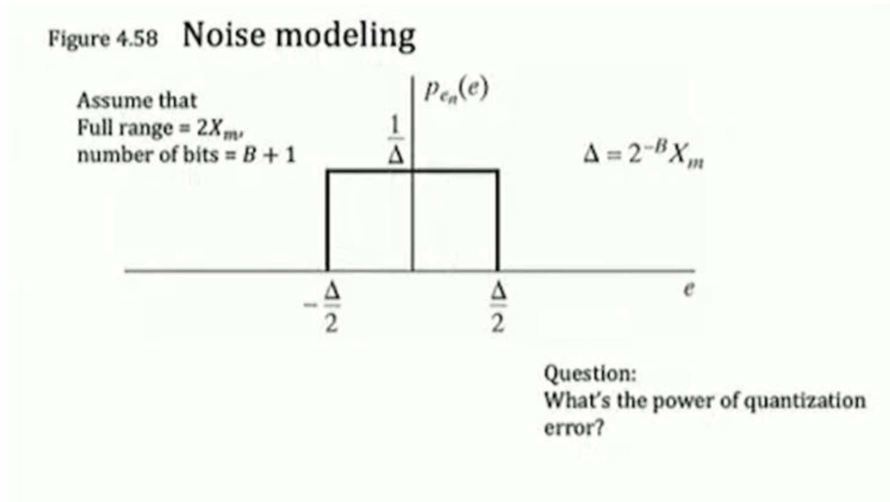
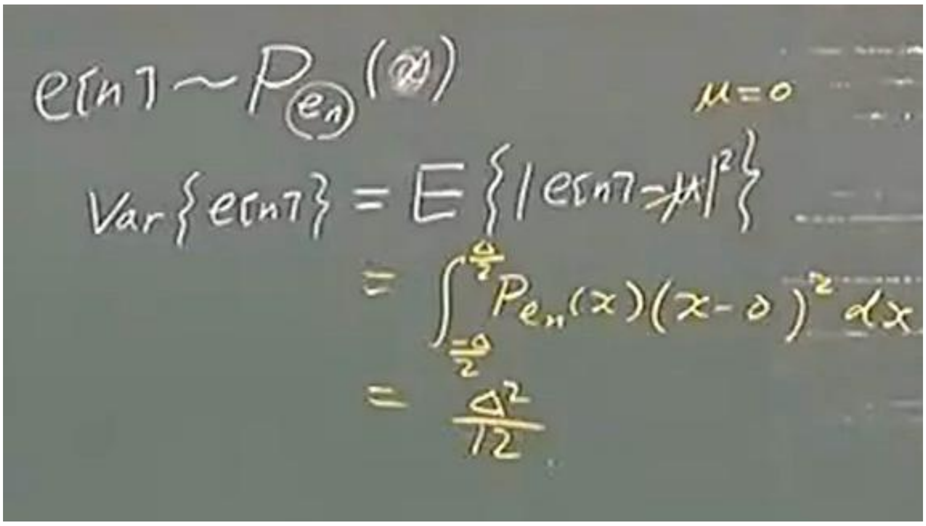


Figure 4.58 Noise modeling
Assume that
Full range = $2X_m$,
number of bits = $B + 1$
$p_{e_n}(e)$
$\frac{1}{\Delta}$
$\Delta = 2^{-B}X_m$
$-\frac{\Delta}{2}$
$\frac{\Delta}{2}$
e
Question:
What's the power of quantization error?


histogram
長條圖.


$e[n] \sim P_{e_n}(x)$
$\mu = 0$
$Var\{e[n]\} = E\{|e[n]-\mu|^2\}$
$= \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} P_{e_n}(x)(x-0)^2\,dx$
$= \frac{\Delta^2}{12}$

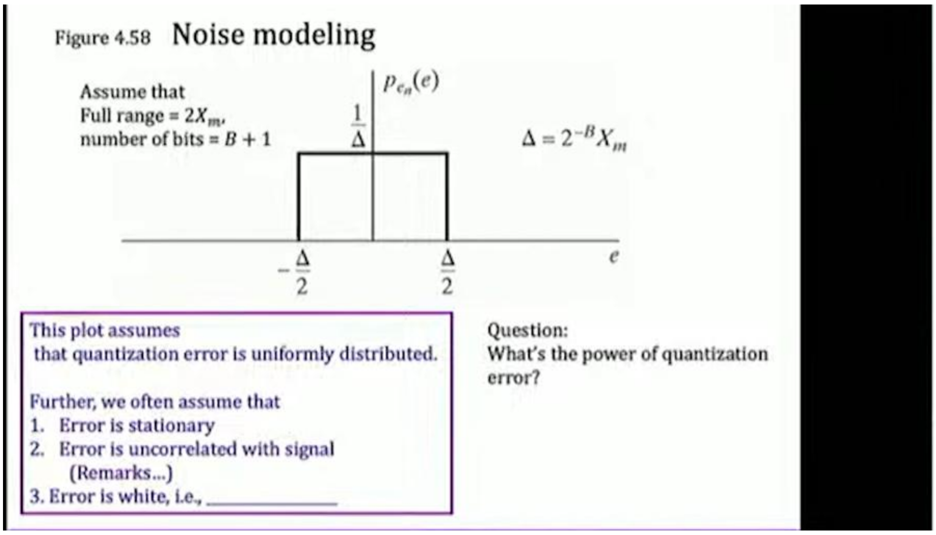
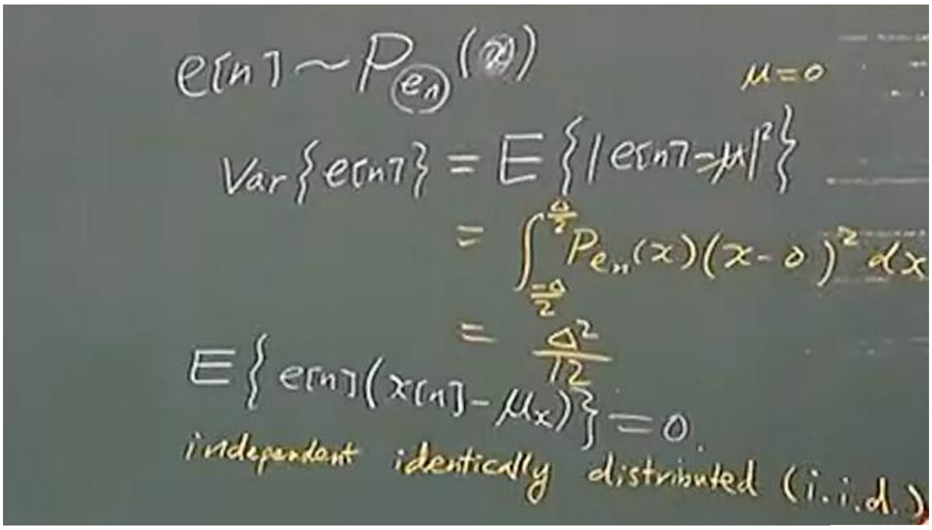
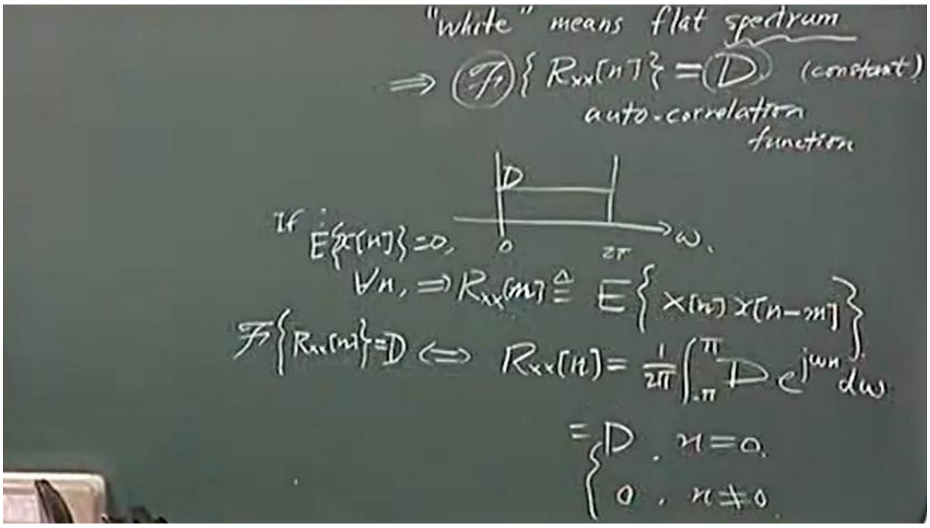
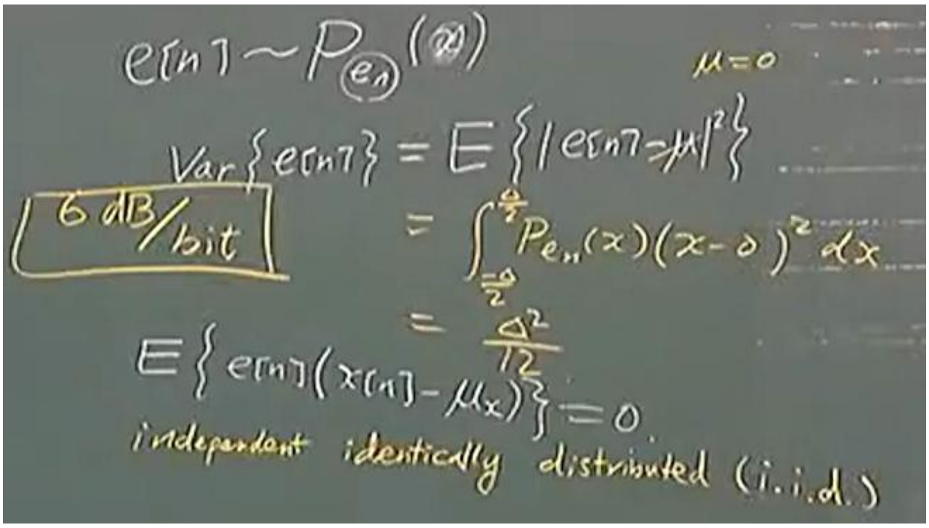
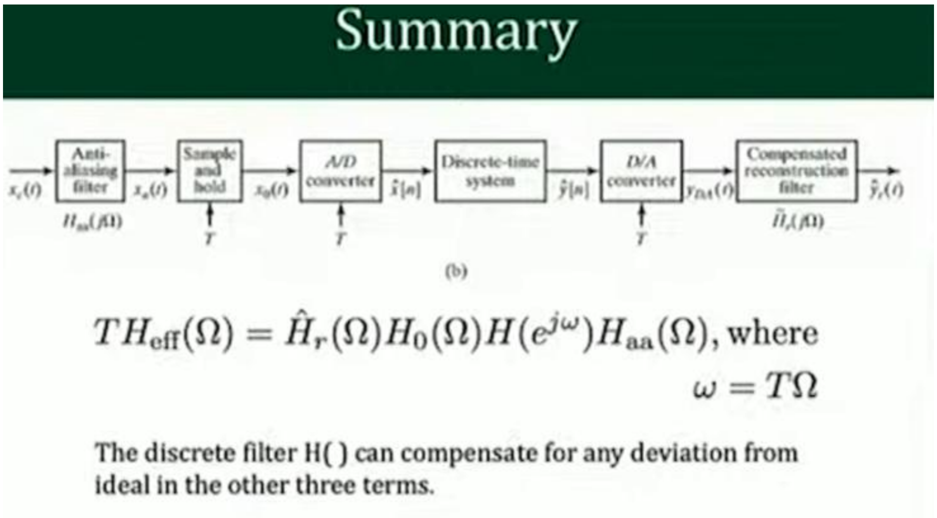


Figure 4.58 Noise modeling
Assume that
Full range = $2X_m$,
number of bits = $B+1$
$p_{e_n}(e)$
$\frac{1}{\Delta}$
$\Delta = 2^{-B}X_m$
$-\frac{\Delta}{2}$
$\frac{\Delta}{2}$
e
This plot assumes
that quantization error is uniformly distributed.
Further, we often assume that
1. Error is stationary
2. Error is uncorrelated with signal
(Remarks...)
3. Error is white, i.e., _____________
Question:
What's the power of quantization error?


$e[n] \sim P_{e_n}(x)$
$\mu = 0$
$Var\{e[n]\} = E\{|e[n]-\mu|^2\}$
$= \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} P_{e_n}(x)(x-0)^2\,dx$
$= \frac{\Delta^2}{12}$
$E\{e[n](x[n]-\mu_x)\} = 0$
independent identically distributed (i.i.d.)


"white" means flat spectrum
auto-correlation function
(constant)


6 dB/bit
independent identically distributed (i.i.d.)


Summary
Anti-aliasing filter
Sample and hold
A/D converter
Discrete-time system
D/A converter
Compensated reconstruction filter
(b)
The discrete filter H( ) can compensate for any deviation from ideal in the other three terms.